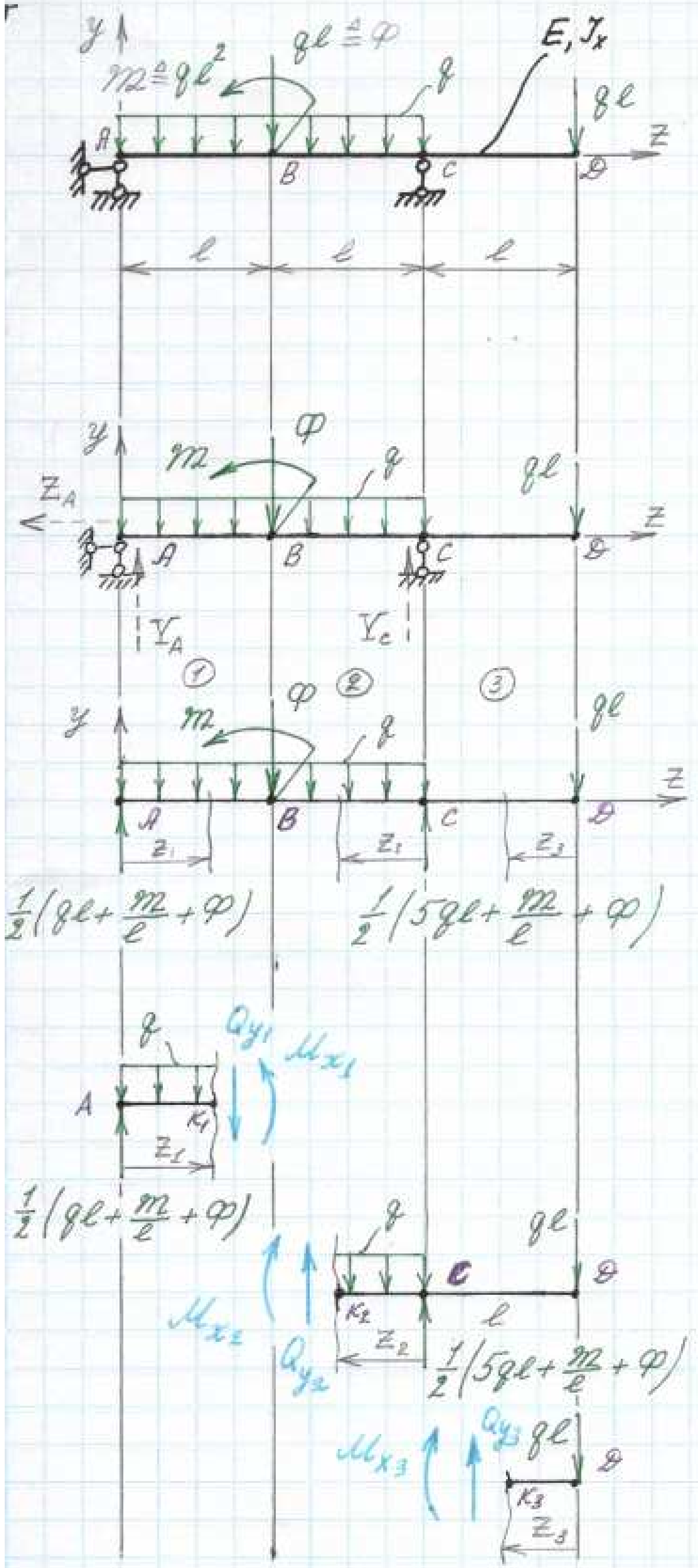

$V_B = ?$ $\quad \theta_B = ?$

Решаем задачу, используя теорему Кастилиано.

$F_B = ql \triangleq \Phi$

$\mathcal{M}_B = ql^2 \triangleq \mathfrak{M}$

Реакции в опорах:

$$\sum \mathcal{M}_A = 0 = -\overset{A-C}{2ql\cdot l} + \overset{B}{\mathfrak{M}} - \overset{B}{\Phi\cdot l} - \overset{D}{ql\cdot 3l} + \overset{C}{Y_C}\cdot 2l$$

$$\Downarrow$$

$$Y_C = \frac{1}{2}\left(5ql - \frac{\mathfrak{M}}{l} + \Phi\right)$$

$$\sum \mathcal{M}_C = 0 = \overset{A-C}{2ql\cdot l} + \overset{B}{\Phi l} + \overset{B}{\mathfrak{M}} - \overset{D}{ql\cdot l} - \overset{A}{Y_A}\cdot 2l$$

$$\Downarrow$$

$$Y_A = \frac{1}{2}\left(ql + \frac{\mathfrak{M}}{l} + \Phi\right)$$

$$\sum F_z = 0 = -Z_A \Rightarrow Z_A = 0$$

Внутренние изгибающие моменты:

$$\sum \mathcal{M}_{K_1} = 0 = -\frac{1}{2}\left(ql + \frac{\mathfrak{M}}{l} + \Phi\right)z_1 + qz_1\frac{z_1}{2} + \mathcal{M}_{x_1} \Rightarrow \mathcal{M}_{x_1} = \frac{1}{2}\left(z_1 ql + z_1\Phi + z_1\frac{\mathfrak{M}}{l} - qz_1^2\right)$$

$$\sum \mathcal{M}_{K_2} = 0 = -\mathcal{M}_{x_2} - qz_2\cdot\frac{z_2}{2} + \frac{1}{2}\left(5ql + \frac{\mathfrak{M}}{l} + \Phi\right)\cdot z_2 - ql(z_2 + l)$$

$$\mathcal{M}_{x_2} = \frac{1}{2}\left(3qlz_2 + \Phi\cdot z_2 - \frac{\mathfrak{M}}{l}z_2 - qz_2^2 - 2ql^2\right)$$

$$\sum \mathcal{M}_{K_3} = 0 = -\mathcal{M}_{x_3} - ql\cdot z_3 \Rightarrow \mathcal{M}_{x_3} = -ql\cdot z_3$$

Потенциальная энергия упругой деформации:

$$U = U^{①} + U^{②} + U^{③} = \int_0^{l_1} \frac{M_{x_1}^2 \cdot dz_1}{2 \cdot E_1 J_{x_1}} + \int_0^{l_2} \frac{M_{x_2}^2 \cdot dz_2}{2 E_2 J_{x_2}} + \int_0^{l_3} \frac{M_{x_3}^2 \cdot dz_3}{2 E_3 J_{x_3}} =$$

$$= \frac{1}{2EJ_x} \cdot \left[\int_0^{l} M_{x_1}^2 \cdot dz_1 + \int_0^{l} M_{x_2}^2 dz_2 + \int_0^{l} M_{x_3}^2 dz_3 \right] =$$

$$= \frac{1}{2EJ_x} \cdot \left[\frac{1}{4} \int_0^{l} \left(z_1 \cdot ql + z_1 \cdot \Phi + z_1 \cdot \frac{\mathfrak{M}}{l} - q z_1^2 \right)^2 \cdot dz_1 + \right.$$

$$+ \frac{1}{4} \int_0^{l} \left(3ql z_2 + \Phi z_2 - \frac{\mathfrak{M}}{l} z_2 - q z_2^2 - 2ql^2 \right)^2 dz_2 +$$

$$\left. + \int_0^{l} q^2 l^2 z_3^2 \, dz_3 \right] =$$

$$= \frac{1}{2EJ_x} \left[\frac{1}{120} \cdot q^2 l^5 + \frac{10}{120} \Phi^2 l^3 + \frac{10}{120} \mathfrak{M}^2 l + \frac{5}{120} q l^4 \Phi + \frac{5}{120} q l^3 \mathfrak{M} + \frac{20}{120} \Phi \mathfrak{M} l^2 + \right.$$

$$+ \frac{31}{120} \cdot q^2 l^5 + \frac{10}{120} \Phi^2 l^3 + \frac{10}{120} \mathfrak{M}^2 l - \frac{15}{120} q l^4 \Phi + \frac{15}{120} q l^3 \mathfrak{M} - \frac{20}{120} \Phi \mathfrak{M} l^2 +$$

$$\left. + \frac{40}{120} q^2 l^5 \right] =$$

$$= \frac{1}{120 \cdot EJ_x} \cdot \left[21 \cdot q^2 l^5 + 10 \cdot \Phi^2 l^3 + 10 \cdot \mathfrak{M}^2 l - 5 q l^4 \Phi + 10 q \mathfrak{M} l^3 \right]$$

Перемещения точки - производные от энергии по соответствующим силам:

$$V_B = \frac{\partial U}{\partial \Phi} = \frac{1}{120 \cdot EJ_x} \left[20 \cdot \Phi l^3 - 5 q l^4 \right] = \frac{1}{120 EJ_x} \left[20 ql \cdot l^3 - 5 q l^4 \right] = \frac{1}{8} \frac{q l^4}{EJ_x} //$$

$$\theta_B = \frac{\partial U}{\partial \mathfrak{M}} = \frac{1}{120 EJ_x} \cdot \left[20 \cdot \mathfrak{M} l + 10 \cdot q l^3 \right] = \frac{1}{120 EJ_x} \cdot \left[20 \cdot q l^2 \cdot l + 10 q l^3 \right] = \frac{1}{4} \frac{q l^3}{EJ_x} //$$

Оба перемещения >0, т.е. по направлению сил Φ и $\mathfrak{M}$

Как вычислялись слагаемые:

$$\frac{1}{4}\int_0^l \left(z_1 q l + z_1 \varphi + z_1 \frac{m}{l} - q z_1^2\right)^2 dz_1 =$$

$$= \frac{1}{4}\int_0^l \left(z_1^2 q^2 l^2 + z_1^2 \varphi^2 + z_1^2 \frac{m^2}{l^2} + q^2 z_1^4 + 2 z_1^2 q l \varphi + 2 z_1^2 q m - \right.$$

$$\left. - 2 z_1^3 q^2 l + 2 z_1^2 \varphi \frac{m}{l} - 2 z_1^3 q \varphi - 2 z_1^3 q \frac{m}{l}\right) dz_1 =$$

$$= \frac{1}{4}\left\{\left(q^2 l^2 + \varphi^2 + \frac{m^2}{l^2} + 2 q l \varphi + 2 q m + 2\frac{\varphi m}{l}\right)\cdot \int_0^l z_1^2\, dz_1 - \right.$$

$$\left. - \left(2 q^2 l + 2 q \varphi + 2 q \frac{m}{l}\right)\int_0^l z_1^3\, dz_1 + q^2 \int_0^l z_1^4\, dz_1\right\} =$$

$$= \frac{1}{4}\left\{\left(q^2 l^2 + \varphi^2 + \frac{m^2}{l^2} + 2 q l \varphi + 2 q m + 2\frac{\varphi m}{l}\right)\frac{l^3}{3} - \right.$$

$$\left. - \left(q^2 l + q\varphi + \frac{q m}{l}\right)\frac{l^4}{2} + q^2 \frac{l^5}{5}\right\} =$$

$$= \frac{1}{4}\left\{\frac{1}{3} q^2 l^5 + \frac{1}{3}\varphi^2 l^3 + \frac{1}{3} m^2 l + \frac{2}{3} q l^4 \varphi + \frac{2}{3} q l^3 m + \frac{2}{3}\varphi m l^2 - \right.$$

$$\left. - \frac{1}{2} q^2 l^5 - \frac{1}{2} q \varphi l^4 - \frac{1}{2} q m l^3 + \frac{1}{5} q^2 l^5\right\} =$$

$$= \frac{1}{4}\left\{\frac{1}{30} q^2 l^5 + \frac{10}{30}\varphi^2 l^3 + \frac{10}{30} m^2 l + \frac{5}{30} q l^4 \varphi + \frac{5}{30} q l^3 m + \frac{20}{30}\varphi m l^2\right\} =$$

$$= \frac{1}{120} q^2 l^5 + \frac{10}{120}\cdot\varphi^2 l^3 + \frac{10}{120} m^2 l + \frac{5}{120} q l^4 \varphi + \frac{5}{120} q l^3 m + \frac{20}{120}\cdot\varphi m l^2$$

$$\frac{1}{4}\int_0^l \left(3ql z_2 + \varphi z_2 - \frac{m}{l} z_2 - q\cdot z_2^2 - 2ql^2\right)^2 dz_2 =$$

$$= \frac{1}{4}\int_0^l \Big(9q^2l^2 z_2^2 + \varphi^2 z_2^2 + \frac{m^2}{l^2} z_2^2 + q^2 z_2^4 + 4q^2l^4 +$$

$$+ z_2^2\, 6ql\varphi - z_2^2\, 6qm - z_2^3 q^2 l - z_2\, 12 q^2 l^3 -$$

$$- z_2^2\, 2\varphi \frac{m}{l} - z_2^3\, 2q\varphi - z_2\, 4ql^2\varphi +$$

$$+ z_2^3\, 2q\frac{m}{l} + z_2\cdot 4qlm + z_2^2\, 4q^2l^2 \Big) dz_2 =$$

$$= \frac{1}{4}\Big\{ 4q^2l^4 \int_0^l dz_2 + \left(4qlm - 4ql^2\varphi - 12q^2l^3\right)\int_0^l z_2\, dz_2 +$$

$$+ \left(9q^2l^2 + \varphi^2 + \frac{m^2}{l^2} + 6ql\varphi - 6qm - 2\varphi\frac{m}{l} + 4q^2l^2\right)\int_0^l z_2^2\, dz_2 +$$

$$+ \left(2q\frac{m}{l} - 2q\varphi - 6q^2 l\right)\int_0^l z_2^3\, dz_2 + q^2\int_0^l z_2^4\, dz_2 \Big\} =$$

$$= \frac{1}{4}\Big\{ 4q^2l^5 + \left(4qlm - 4ql^2\varphi - 12q^2l^3\right)\frac{l^2}{2} + \Big(9q^2l^2 + \varphi^2 +$$

$$+ \frac{m^2}{l^2} + 6ql\varphi - 6qm - 2\varphi\frac{m}{l} + 4q^2l^2\Big)\frac{l^3}{3} +$$

$$+ \left(2q\frac{m}{l} - 2q\varphi - 6q^2 l\right)\frac{l^4}{4} + \frac{1}{5}q^2l^5 \Big\} =$$

$$= \frac{31}{120}q^2l^5 + \frac{10}{120}\varphi^2 l^3 + \frac{10}{120}m^2 l - \frac{15}{120}ql^4\varphi + \frac{15}{120}ql^3 m - \frac{20}{120}\varphi m l^2$$

$$\int_0^l q^2 l^2 z_3^2 \, dz_3 = q^2 l^2 \int_0^l z_3^2 \, dz_3 = q^2 l^2 \frac{l^3}{3} = \frac{1}{3} q^2 l^5 =$$

$$= \frac{40}{120} \cdot q^2 l^5$$